Use of Trilinos for Stellar Hydrodynamics

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Tuesday 3rd March, 2015

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Challenges of Stellar Interiors

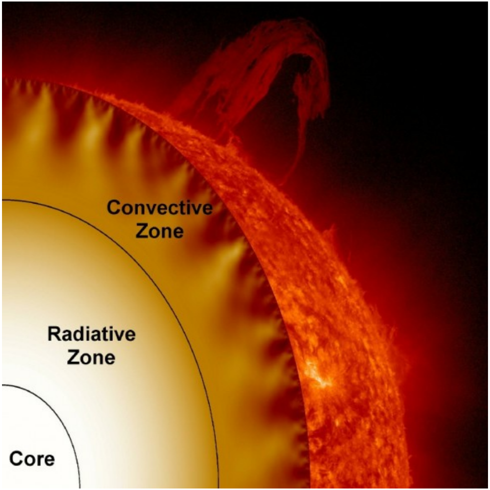
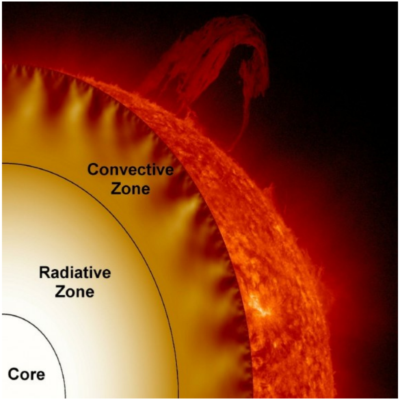


Image source: http://solarscience.msfc.nasa.gov/images/cutaway.jpg

Challenges of Stellar Interiors

Stellar physics involves many complex processes characterised by vastly different:

- Time scales (solar values):
    - $\tau_{\text{therm}} \sim 10^7$ years,
    - $\tau_{\text{nucl}} > 10^9$ years,
    - $\tau_{\text{Dyn}} \sim 30$ mins
- Length scales,
    - $H_P = \frac{dr}{d \ln P}$
    - $10^{-5} R_* \leq H_p \leq R_*$
- Mach numbers,
    - $M_s = \frac{|\vec{u}|}{C_s}$,
    - $10^{-10} \leq M_s \leq 1$



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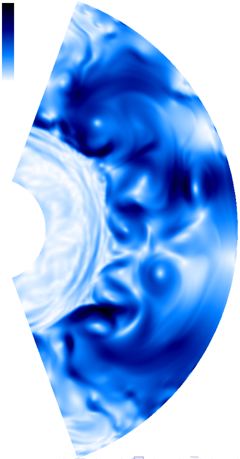


1D Stellar Evolution

- Stellar evolution dependent on 1D, spherically symmetric simulations
- Parametrise multidimensional phenomena:
  - Shear Mixing
  - Turbulent convection (Mixing length theory)
  - Accretion
  - Rotation
- Many free parameters-hinder predictive capabilities
- However, it is these results we must link to observations

3D Stellar Physics

- Test the 1D formalism
  - Improve predictive capability
- Lots of data becoming available (Gaia, CoRoT, Kepler)
- Existing work often uses explicit or Boussinesq or anelastic approximations
  - Inappropriate for modelling entire stellar interior
  - Explicit methods severely restrict time of simulation
- MUSIC offers fully compressible, implicit, hydrodynamics

Spatial Discretisation

$$\frac{\partial}{\partial t}\rho = -\nabla \cdot (\rho \vec{u})$$
$$\frac{\partial}{\partial t}\rho e = -\nabla \cdot (\rho e \vec{u}) - P\nabla \cdot \vec{u} - \nabla \cdot (\chi \nabla T)$$
$$\frac{\partial}{\partial t}\rho \vec{u} = -\nabla \cdot (\rho \vec{u} \otimes \vec{u}) - \nabla P + \rho \vec{g}$$

- Finite volume discretisation
- Total variation diminishing
- Second order in space

$$P = P(\rho, e)$$
$$T = T(\rho, e)$$
$$\chi = \frac{16\sigma T^3}{3\kappa\rho}$$
$$\kappa = \kappa(T, \rho)$$

Temporal Discretisation

- Crank-Nicolson, 2nd order in time
- Use Newton-Raphson method to solve non-linear problem
- Linear Solve-Trilinos, JFNK
- Preconditioner, approximate semi-implicit system
- Semi-implicit system-Trilinos, Matrix based.

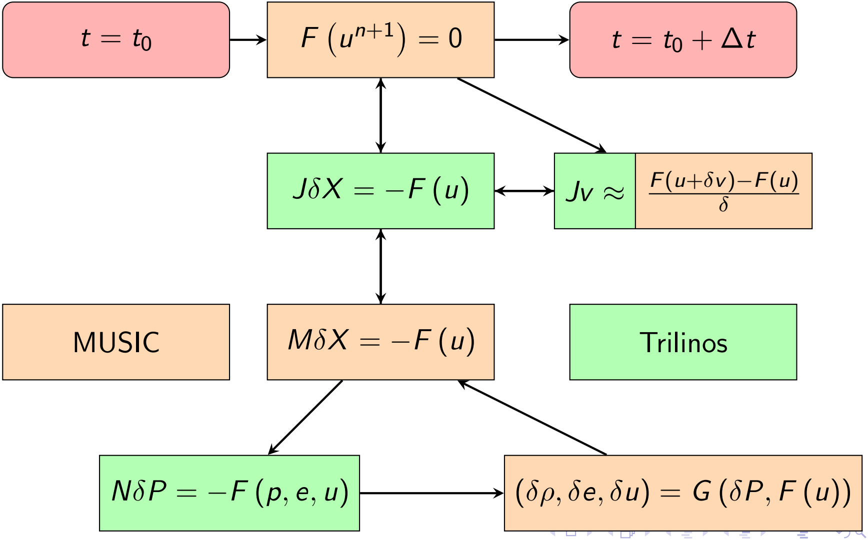
Preconditioner

- Use approximate solution of full implicit system as preconditioning operator.
- Purpose written semi-implicit (SI) scheme:
  - Linearisation, advection treated explicitly.
  - First order in time&space.
  - Treats sound waves, and optionally thermal diffusion implicitly.
  - Stability based on $\frac{\Delta x}{|\vec{u}|}$, not, $\frac{\Delta x}{C_s+|\vec{u}|}$
  - Designed to be Mach number independent.

Preconditioner

- Use approximate solution of full implicit system as preconditioning operator.
- Purpose written semi-implicit (SI) scheme.
- Resulting linear problem smaller and more sparse.
- Solve using GMRES, ML-Preconditioner

Typical Time Step



$t = t_0$

$F\left(u^{n+1}\right) = 0$

$t = t_0 + \Delta t$

$J\delta X = -F\left(u\right)$

$Jv \approx \frac{F(u+\delta v)-F(u)}{\delta}$

MUSIC

$M\delta X = -F\left(u\right)$

Trilinos

$N\delta P = -F\left(p, e, u\right)$

$\left(\delta\rho, \delta e, \delta u\right) = G\left(\delta P, F\left(u\right)\right)$

Mach Number Independence

- Key numerical challenge
- Motivation behind time-implicit methods
- Calibrate MUSIC using ideal gas problems
- Seek range, $10^{-6} \leq \mathrm{M_s} \leq 10^{-1}$

2D Vortex Advection

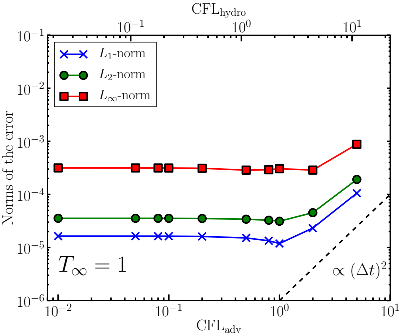
- Problem definition:

$$\rho = (T_\infty + \delta T)^{\frac{1}{\gamma-1}} \qquad e = \frac{\rho^{\gamma-1}}{\gamma-1}.$$

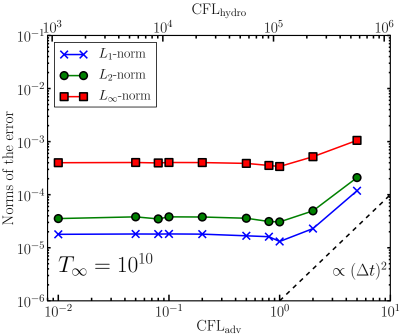
$$u = u_\infty + \delta u \qquad v = v_\infty + \delta v$$

- Vary Mach number by fixing u_∞,v_∞, and varying T_∞.
- Calculate L_1,L_2,L_∞ norms on velocity
- Simulate problem for range of fixed timesteps.

2D Vortex Problem - Errors



$\mathrm{M_s} = 10^{-1}$



$\mathrm{M_s} = 10^{-6}$

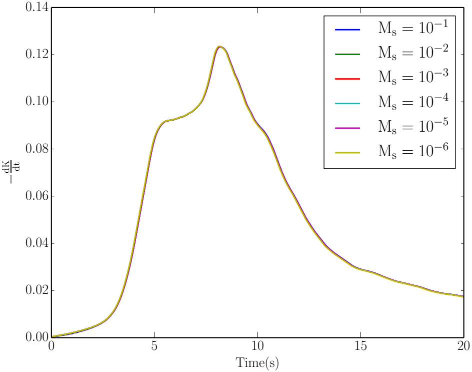
Taylor Green Vortex Decay

- Problem definition:

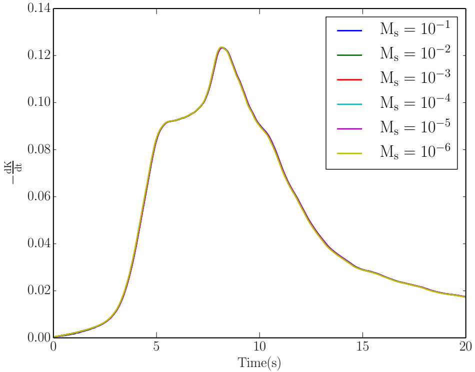
$$\begin{aligned}
u_x(x,y,z) &= u_0 \sin x \cos y \cos z, \\
u_y(x,y,z) &= -u_0 \cos x \sin y \cos z, \\
u_z(x,y,z) &= 0 \qquad \rho = 1.0, \\
p(x,y,z) &= p_0 + \tfrac{1}{16}\rho_0 u_0^2 (2 + \cos 2z)(\cos 2x + \cos 2y)
\end{aligned}$$

- Vary Mach number by fixing $u_0 = 1.0$, and varying p_0.
- Investigate Mach number dependency, by measuring $\frac{dK}{dt}$

Taylor Green Vortex Decay



Taylor Green Vortex Decay



Maximum difference in peak dissipation: 0.2%

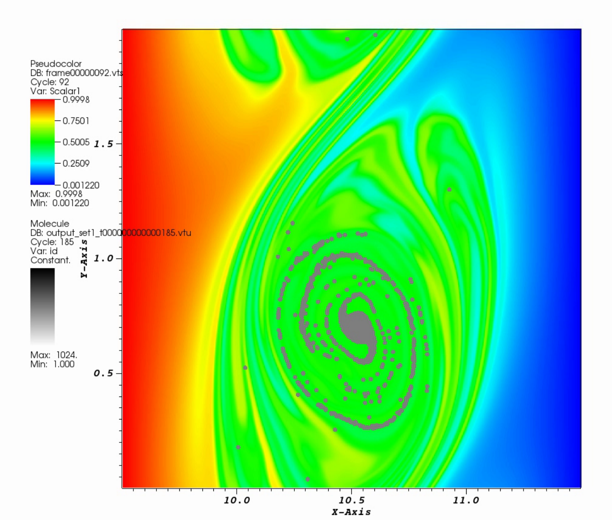
Trilinos Settings

- useNewPerturbation=false
    - $\lambda = 10^{-7}$
- But very code specific, see also:
    - Non-linear settings
    - Scaling/Normalisation
- Probably degenerate problem

Shear Mixing

- Important process in terms of:
  - Stellar Evolution
  - Extension of stellar life time
  - Linking to observations
- Try to quantify mixing using:
  - Tracer Particles
  - Passive Scalars (additional linear solve!)

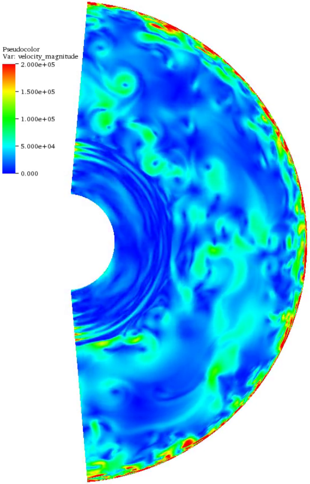
Scalar-Particle Test



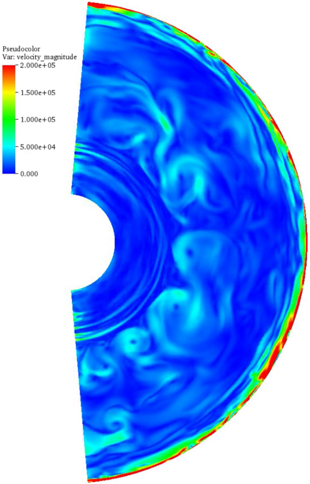
Accretion(C. Geroux)

- Modelling of interaction of star and environment
- Consider already formed star
- Accretion modelled can be described as:
    - Bursts/Short Term
    - Hot
- Effectively simulated as inflow boundary condition.

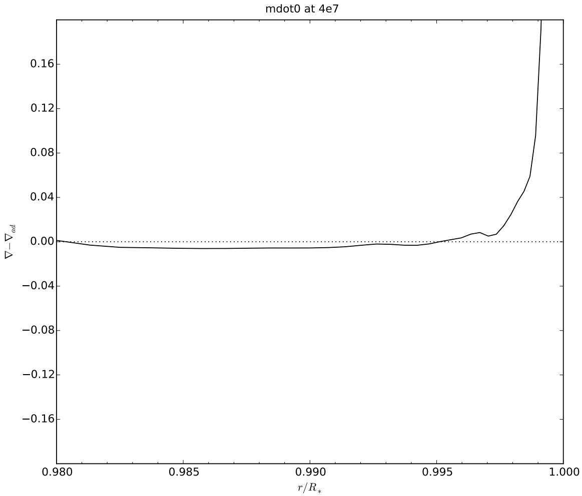
Accretion Control Run



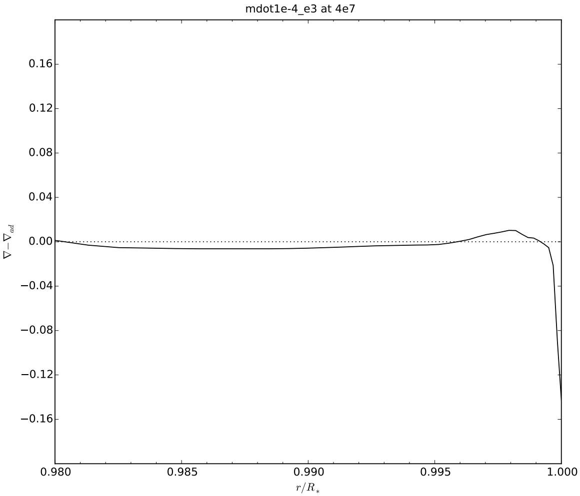
Hot Accretion



Convective Stability Control Run



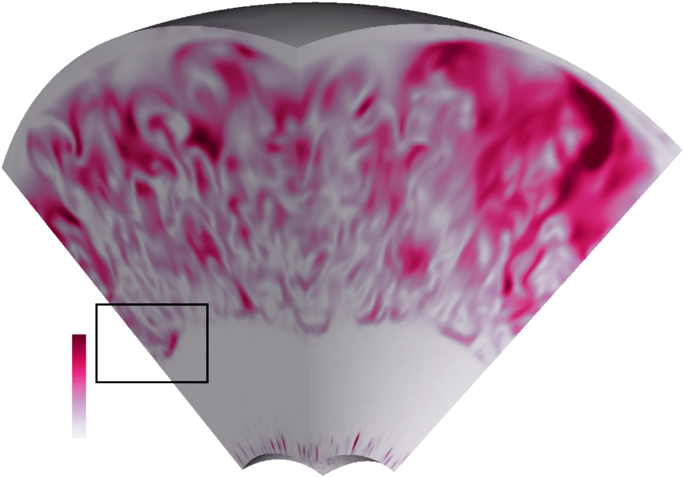
Convective Stability with Accretion



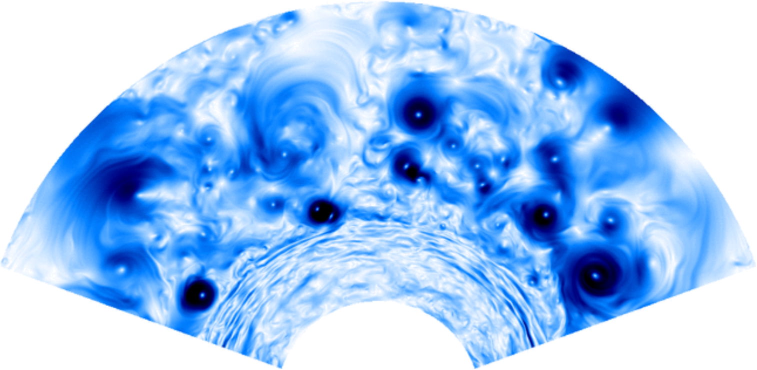
Convective-Radiative Zone Interaction (J. Pratt)

- Convective flows can displace fluid from convectively unstable to stable zones
- Penetration refers to a permanent displacement of this boundary
- Overshooting is a short-time ballistic process
- Convective-Radiative boundary can be important for:
  - Chemical Mixing
  - Angular Momentum Transport
  - Dynamo Processes

Overshooting 3D



Overshooting 2D



Questions?